ACKNOWLEDGMENT

This study was performed under Grant CPE 7908893, National Science Foundation.

NOTATION

C_D = drag coefficient
d = particle diameter
$F(\epsilon)$ = a function appearing in Eq. 4
g = acceleration resulting from gravity
M = μ_m/μ_ℓ
N_{Re} = Reynolds number defined by Eq. 2
$N_{Re,m}$ = Reynolds number for a suspension defined by Eq. 9
$N_{Re,t}$ = Reynolds number based on terminal velocity, defined by Eq. 6
R = ρ_s/ρ_ℓ
U_o = superficial liquid velocity
U_R = v_r/U_t
U_t = terminal velocity of a particle in an infinite medium
v_r = relative velocity between liquid and particles

Subscripts

i = ith particle species
ℓ = liquid
m = mixture or suspension
s = solid

Greek Letters

α = $C_D(N_{Re,t})^2$
β = functional form in Eq. 7
ϵ = porosity
ρ = density
μ = viscosity

LITERATURE CITED

Garside, J., and M. R. AlDibouni, "Velocity-Voidage Relationships for Fluidization and Sedimentation in Solid Liquid Systems," *Ind. Eng. Chem. Proc. Des. Dev.*, **16**, 206 (1977).

Ishii, M., and N. Zuber, "Drag Coefficient and Relative Velocity in Bubbly, Droplet or Particulate Flows," *AIChE J.*, **25**, 843 (1979).

Leva, M., *Fluidization*, McGraw-Hill, New York (1959).

Masliah, J. H., "Hindered Settling in a Multi-species Particle System," *Chem. Eng. Sci.*, **34**, 1166 (1979).

Richardson, J. F., and R. A. Meikle, "Sedimentation of Uniform Fine Particles and of Two Component Mixtures of Solids," *Trans. Inst. Chem. Engrs.*, **39**, 348 (1961).

Richardson, J. F., and W. N. Zaki, "Sedimentation and Fluidization," *Trans. Inst. Chem. Engrs.*, **32**, 35 (1954).

Thomas, D. G., "Transport Characteristics of Suspensions: A Note on the Viscosity of Newtonian Suspensions of Uniform Spherical Particles," *J. Colloid. Sci.*, **20**, 267 (1965).

Wallis, G. B., *One Dimensional Two Phase Flow*, McGraw Hill, New York (1969).

Wilhelm, R. H., and M. Kwauk, "Fluidization of Solid Particles," *Chem. Eng. Prog.*, **44**, 201 (1948).

Manuscript received October 7, 1982; revision received June 3, and accepted June 23, 1983.

Electroconvective Enhancement of Mass or Heat Exchange Between a Drop or Bubble and Surroundings in the Presence of an Interfacial Tension Gradient

L. S. CHANG and J. C. BERG

Department of Chemical Engineering BF-10
University of Washington
Seattle, WA 98195

Experimental evidence suggests (Thornton, 1968; Bailes and Thornton, 1971; Harker and Ahmadzadeh, 1974; Bailes, 1981) and computations appear to confirm (Morrison, 1977; Griffiths and Morrison, 1979; Chang et al., 1982; Chang and Berg, 1983) that electric fields may, under certain circumstances, lead to significant enhancements in the mass (or heat) exchange rate between drops or bubbles and a surrounding liquid medium. The premise of the computational models is that such enhancement may be traced to electric-field-induced circulation in the drop, as first described by G. I. Taylor (1966). When the drop is stationary with respect to the external medium and the electric field is uniform, the circulation takes the form of a symmetrical pair of tori inside the drop, with surface flow directed either from the poles toward the equator or vice versa, depending on the electrical properties of the system. For the situation more likely to exist in practice, in which the drop or bubble translates by gravity and the electric field is oriented parallel to the translation, the flow becomes a hybrid between the single torus, Hadamard-Rybczynski circulation and the electrohydrodynamic circulation, as described by Taylor (Chang et al., 1982; Chang and Berg, 1983). The torus on the rear side of the drop is reduced in size and may be absent altogether. Under the latter conditions, no significant electroconvective enhancement of exchange rates may be anticipated. The extent of flow modification by the electric field may be conveniently characterized in terms of the dimensionless parameter W (Chang et al., 1982):

$$W = \frac{4V(1 + X)}{U_0} \qquad (1)$$

where V is the maximum electrically-induced interfacial velocity in the corresponding Taylor problem and is given by:

$$V = \frac{9E^2 a \epsilon_2 (1 - \epsilon_1 S_1/\epsilon_2 S_2)}{10(2 + S_1/S_2)^2 \mu_1 (1 + X)} \qquad (2)$$

L. S. Chang is presently with the Union Carbide Corp.
Correspondence concerning this paper should be addressed to J. C. Berg.

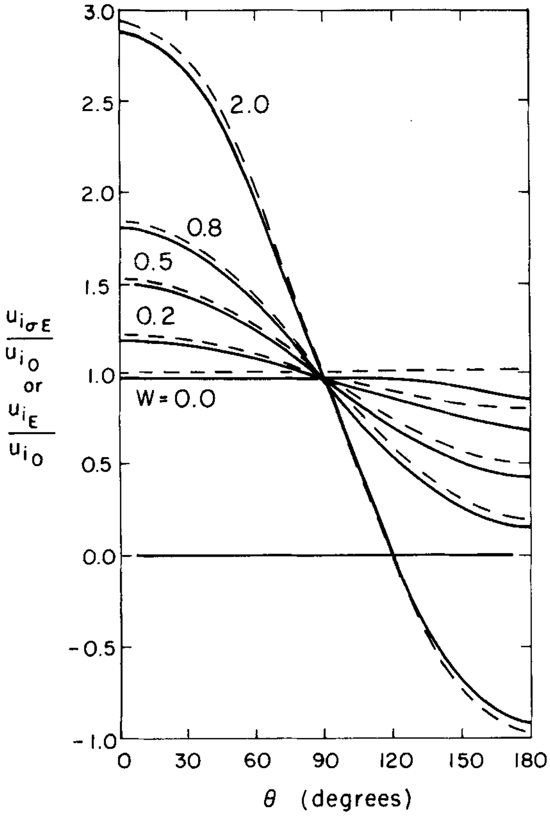

Figure 1. Ratios of interfacial velocities as a function of the polar angle measured from the frontal stagnation point. The dashed lines are for u_{i_E}/u_{i_0}, whereas the solid lines are for $u_{i_{\sigma E}}/u_{i_0}$. Numerical values correspond to the following conditions: $a = 1.0 \times 10^{-5}$ m; $K = 4.0 \times 10^{-7}$ m; $T = 293.15$ K; $U_0 = 4.58 \times 10^{-4}$ m/s^{-1}; $\mu_1 = 7.0 \times 10^{-4}$ Pa·s; $D_1 = 1.53 \times 10^{-10}$ m^2·s^{-1}; $C_\infty = 0.05$ mmol·m^{-3}.

U_o is the Hadamard-Rybczynski terminal velocity, which is unaffected by the electric flow as long as creeping flow conditions prevail. For creeping flow, the condition for the existence of the second torus is $|W| > 1$. For pure solvent systems, W takes the form:

$$W = \Phi(\text{physical properties})\,\frac{E^2}{a} \tag{3}$$

where E is the electric field intensity. W values for moderate electric field strengths and based upon the nominal electrical properties of the solvents have been evaluated for many systems and for most have been found to be much less than unity. Systems *can* be found, however, for which W is large, and it should be noted that the electrical properties upon which W depends, particularly the resistivities, are highly sensitive to the presence of traces of charge-bearing impurities. Of particular interest is the fact that large W systems may be created by the use of very small amounts of oil-soluble electrolyte species.

Computations of the transfer efficiency as a function of W and related parameters have been carried out for the high Peclet number (thin concentration boundary layer), creeping flow case of a drop or bubble translating in an electric field (Chang et al., 1982). In addition to the above-stated limitations, these results suffer from the fundamental shortcoming of not accounting for interfacial tension gradients around the drop periphery. The mass (or heat) transfer process will generally lead to the development of such gradients (Carleson and Berg, 1983), but it is the presence of traces of surfactant (either as the transferring solute, an additive used to increase W, or as a contaminant) that may lead to major modifications of the flow structure. We have recently obtained stream functions for the drop and the continuous phase for the case of steady steady creeping flow in the presence of gravity, an imposed electric field and an arbitrary axisymmetric interfacial tension gradient (Chang and Berg, 1983a). From these we have determined the quasisteady interfacial distribution of surfactant under various conditions and the corresponding interfacial velocity distribution and terminal velocity. Numerical results are presented for the case of a very small air bubble rising in a liquid of moderate viscosity under various conditions. We have found that, depending upon the electrical properties of the system, the surfactant will tend to concentrate along the interface to produce quasistagnant zones either at the two poles of the drop or near its equator. The present note seeks to examine the consequences of interfacial tension gradients on convective mass (or heat) exchange rates. In particular, it is of interest to determine if the W criterion for assessing the significance of electric field effects still holds in the presence of interfacial tension gradients.

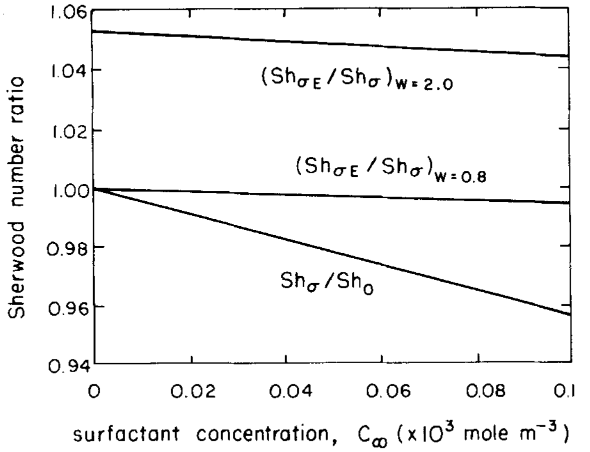

Figure 2. Sherwood number ratios as a function of surfactant concentration for the specific case shown in Figure 2.

We consider here again the case of a small, spherical gas bubble, from which a surfactant species is slowly and quasisteadily absorbed by the continuous liquid phase. For such a case, the thin concentration boundary layer approximation is valid, and in accord with the method of Lochiel and Calderbank (1964), the appropriate Sherwood number is given by:

$$Sh = (2/\pi)^{1/2} Pe^{1/2} \left(\int_0^\pi (u_i/U) \sin^2\theta \, d\theta \right)^{1/2} \tag{4}$$

Equation 4 is valid if the Peclet number is large ($\gtrsim 100$), the viscosity ratio $X \lesssim 2$ and there is negligible recirculation ($W \lesssim 1$).

Equation 4 requires the interfacial velocity distribution under the desired conditions, and for present purposes of comparison, these would correspond to the presence and absence of an interfacial tension gradient (as caused by the presence of a given type and amount of surfactant) for various relative electric field strengths, i.e., various values of W. Some examples of such interfacial velocity distributions are shown in Figure 1 for the specific bubble system considered by Chang and Berg (1983a). The surfactant concentration if $C_\infty = 0.05$ mol·m^{-3}. The curve for $W = 2$ represents a situation in which there is significant recirculation; thus, the method of Chang et al. (1982) is used to compute the Sherwood number. Interfacial velocity is seen to vary sharply with electric field intensity, even for values of W less than unity. The influence of the interfacial tension gradient can be seen by comparing the ratio $u_{i_{\sigma E}}/u_{i_0}$ to u_{i_E}/u_{i_0} for given values of W. The most significant effect of the modest interfacial tension gradients developed in this example is the large reduction of interfacial velocity in the rear stagnation region of the bubble for low values of W. A modest and approximately uniform reduction occurs over the rest of the bubble surface. Velocity reduction at the rear of the bubble is moderated as W approaches unity, because the electric field effect reduces the intensity of the convergent flow there. When W surpasses unity, the flow separates, and its reversal in the rear part of the bubble is evident in the curve for $W = 2.0$.

When interfacial velocity distributions of the type shown in

Figure 1 are substituted into Eq. 4, results of the type shown in Figure 2 are obtained. The curve of Sh_σ/Sh_0 vs. surfactant concentration shows the reduction in mass transfer coefficient caused by the presence of the interfacial tension gradient in the absence of an electric field. The curve of $Sh_{\sigma E}/Sh_\sigma$ for $W = 0.8$ shows that the electric field effects are essentially negligible below a W value of unity, whereas for $W = 2.0$, quite substantial increases are observed despite the interfacial tension gradients. These results closely parallel those obtained earlier for the electric field effects on transfer rates in the absence of interfacial tension gradients. Thus it appears that the use of traces of surfactant additives (for the purpose of increasing W) or the presence of traces of surfactant contamination will not alter significantly the electroconvective enhancements predicted earlier for mass (or heat) transfer in the presence of electric fields.

ACKNOWLEDGMENTS

This work was supported in part by grants from the National Science Foundation (CPE-9720830) and the Occidental Research Corp.

NOTATION

a = droplet radius
C_∞ = surfactant concentration far away from droplet
D = mass diffusivity
E = uniform electrical field intensity
K = adsorption coefficient for the surfactant
k_L = mass transfer coefficient
Pe = Peclet number, $2Ua/D$
S = electrical resistivity
Sh = Sherwood number, $k_L(2a)/D$
U = terminal velocity of droplet
u_i = interfacial velocity
V = electrically-generated characteristic speed
$W = 4V(1 + X)/U$
X = viscosity ratio, μ_2/μ_1

Greek Letters

ϵ = permittivity
θ = polar angle from frontal stagnation point
μ = viscosity

Subscripts

1 = continuous phase
2 = drop phase
E = electrified, surfactant-free system
0 = nonelectrified, surfactant-free system
σ = surfactant-containing system
σE = electrified, surfactant-containing system

LITERATURE CITED

Bailes, P. J., and J. D. Thornton, "Electrically-Augmented Liquid-Liquid Extraction in a Two-Component System. I: Single Droplet Studies," *Proc. Int. Solvent Extr. Conf.*, **2**, 1431 (1971).

Bailes, P. J., "Solvent Extraction in an Electrostatic Field," *Ind. Eng. Chem. Process Des. Dev.*, **20**, 564 (1981).

Carleson, T. E., and J. C. Berg, "Marangoni and Electric Field Effects upon Mass Transfer to Translating Drops," *Chem. Eng. Commun.* (1983).

Chang, L. S., T. E. Carleson, and J. C. Berg, "Heat and Mass Transfer to a Translating Drop in an Electric Field," *Int. J. Heat Mass Transfer*, **25**, 1023 (1982).

Chang, L. S., and J. C. Berg, "Fluid Flow and Transfer Behavior of a Drop Translating in an Electric Field at Intermediate Reynolds Number," *Int. J. Heat Mass Transfer*, **26**, 823 (1983a).

———, "The Effect of Interfacial Tension Gradients on the Flow Structure of Single Drops or Bubbles Translating in an Electric Field," submitted to *AIChE J.*

Griffiths, S.K., and F. A. Morrison, Jr., "Low Peclet Number Heat and Mass Transfer from a Drop in an Electric Field," *Trans. ASME*, **101**, 484 (1979).

Harker, J. H., and J. Ahmadzadeh, "The Effect of Electric Fields on Mass Transfer from Falling Drops," *Int. J. Heat Mass Transfer*, **17**, 1219 (1974).

Lochiel, A. C., and P. M. Calderbank, "Mass Transfer in the Continuous Phase around Axisymmetric Bodies of Revolution," *Chem. Eng. Sci.*, **19**, 471 (1964).

Morrison, F. A., "Transient Heat and Mass Transfer to a Drop in an Electric Field," *J. Heat Transfer*, Ser. C, **99**, 269 (1977).

Taylor, G. I., "Studies in Electrohydrodynamics. I: The Circulation Produced in a Drop by an Electric Field," *Proc. R. Soc.*, **291A**, 159 (1966).

Thornton, J. D., "The Application of Electrical Energy to Chemical and Physical Rate Processes," *Rev. Pure Appl. Chem.*, **18**, 197 (1968).

Manuscript received July 26, 1983; revision received November 21 and accepted December 5, 1983.